Optimum force magnitude for orthodontic tooth movement: A mathematic model

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The aim of this study was to develop a mathematic model to describe the relationship between magnitude of applied force and rate of orthodontic tooth movement. Initially, data were extracted from experimental studies in dogs (beagles), in which controlled, standardized forces were used to move mandibular second premolars distally. Curve-fitting by nonlinear regression analysis provided an equation describing the relationship between force magnitude and rate of tooth movement in beagles. A similar equation was subsequently used to analyze the limited available data from the literature on human canine retraction. The maximum rates of tooth movement in humans and dogs are very similar. A threshold for force magnitude that would switch on tooth movement could not be defined. The model showed that a wide range of forces can be identified, all of which lead to a maximum rate of tooth movement. (Am J Orthod Dentofacial Orthop 2004; 125:71-7)

Optimal orthodontic treatment requires a mechanical input that leads to a maximum rate of tooth movement with minimal irreversible damage to the root, periodontal ligament, and alveolar bone.[1] Although other biologic indicators, such as cellular response, tissue damage, pain, and the tendency for relapse are important, literature pertinent to the efficiency of orthodontic treatment modalities mainly focuses on the relationship between orthodontic force magnitude and the rate of tooth movement during active treatment. Discussions regarding this issue are generally based on the theories of Quinn and Yoshikawa,[2] who described 4 alternative models for this relationship (Fig 1).

The first model supposes an on/off switch that is switched on at a certain force level (Fig 1, *A*). All forces above this threshold will lead to the same rate of tooth movement. Several studies support this model. Owman-Moll et al[3] found no difference in tipping movement of human premolars with forces of 50 cN and 100 cN. Iwasaki et al[4] found that, in humans, effective tooth movement can be produced with low forces, and that higher forces do not necessarily lead to faster tooth movement. Animal experiments have shown that the maximum rate of tooth movement was similar for a wide range of forces; this indicates that, even with light forces, maximal biologic response could be reached.[5-8] Other studies, however, do not support this model.[9]

In the second model (Fig 1, *B*), a force threshold is also indicated. With forces above the threshold, a linear dose-response relationship is assumed. Several studies[10-14] have shown that higher forces were generally more efficient in moving teeth, and they support this model. However, the forces used in these studies were within a quite narrow range. Up to now, no study across a wide range of continuous forces is available that favors this model.

In the third model (Fig 1, *C*), forces above a certain threshold are necessary to induce movement. A dose-response relationship exists in a lower force range up to a certain level. Then a plateau is reached, and a further increase of force leads to a decrease in the rate of tooth movement, until it ceases completely. This model agrees with the differential force theory proposed by Begg.[15] According to Begg's theory, low forces should be applied for space closure, whereas high forces could be used for anchorage of the segment. These phenomena might be related to the induction of hyalinization in the periodontal ligament. Other studies[10,16-19] support this model. They show maximal canine movement in humans with a limited force range and indicate that, with force values below this range, practically no

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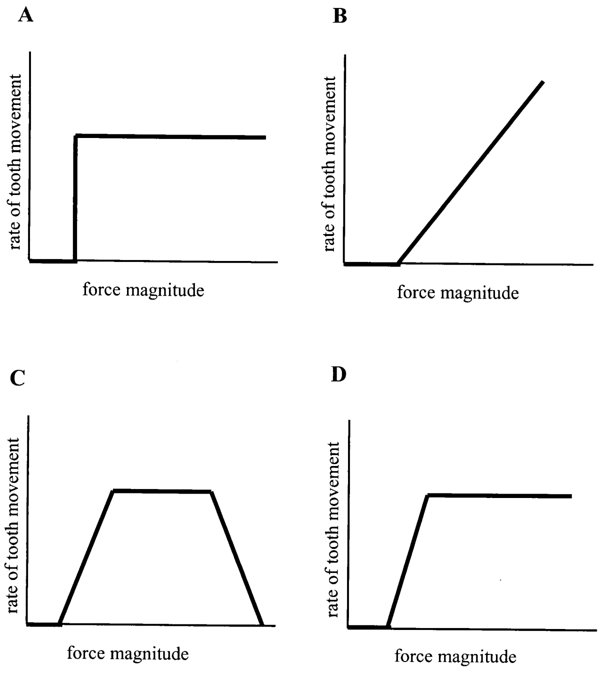

Fig 1. A-D, The 4 models of relationship between force magnitude and velocity of tooth movement, as proposed by Quinn and Yoshikawa.[2]

movement of the canine takes place. Storey and Smith[10] proposed that by increasing the force beyond this range, the rate of tooth movement decreases and finally ceases completely. So far, no study has been published to support these ideas. From a biologic point of view, however, it is feasible that very large forces will slow the recruitment or the differentiation of cells and will cause tissue hyalinization. Both processes will hamper tooth movement and affect cell-matrix interactions.[20]

The last model (Fig 1, *D*) generally resembles the third, but it lacks the decreasing part. Several authors[12,21,22] report data supporting this model. They found no measurable increase in rate of tooth movement above certain force levels. Some animal experiments also support this opinion.[23]

Quinn and Yoshikawa[2] concluded that the last model was the best supported by experimental and clinical data, but their reasoning might be hampered by the lack of available experimental data on the use of high forces.

Recently, a thorough systematic study of the literature pertaining to the efficiency of tooth movement in human subjects, performed by our group,[9] indicated that no optimal force magnitude, or more accurately no optimal pressure magnitude, could be defined. The main problems encountered were related to the inability to estimate stresses or stress distribution in the periodontal ligament, the lack of control of bodily or tipping movement, the variation in follow-up periods, and large individual variations. We have tried, using data from the literature on human studies, to explore several mathematic models to evaluate the relationship between force magnitude and velocity of tooth movement. The models all have the problem of an inadequate fit to biologic data, or they could not indicate the range of optimal force. Thus, the results from the above-mentioned mathematic models were not conclusive.[9]

The problems mentioned above were largely overcome in a series of animal studies in which the effects of different standardized orthodontic forces on mandibular second premolars of beagles were studied over a long period.[5-8] The purpose of the present study was to develop a mathematic model based on data from these standardized animal experiments and to test this model with the data from clinical research.

MATERIAL AND METHODS

Data retrieval

The choice for the data used in the present study was based on a systematic literature review by our group.[9] This literature search covered the period 1966-2001 and found 161 sources dealing with experimental animal studies on the relationship between orthodontic force and subsequent tooth movement. Only 6 of these articles describe the use of standardized orthodontic appliances with predetermined constant forces for long-term bodily tooth movement (1 in monkeys,[24] 5 in beagles[5-8,25]). The 5 articles on beagles reported on experimental distal movement of mandibular second premolars. One of these articles[25] did not provide enough data to be used for the mathematic model. The other 4 studies[4-7] were performed by our group at the Department of Orthodontics and Oral Biology, University of Nijmegen, The Netherlands. The combined data of these 4 studies were retrieved from 146 series of measurements, and all were expressed as applied force (cN) and velocity of tooth movement (millimeters per week).

The experimental setup of the 4 studies was as follows. In young adult male beagles, the mandibular third premolars were extracted. The orthodontic appliance was placed 16 weeks after the extractions. The construction of the appliance was such that the second premolars were only allowed to move distally. Forces of 10, 25, 50, 100, 200, 300, 600, and 1200 cN were used in a split-mouth design, with different forces on the left and right sides. In 2 studies,[5,6] the mandibular canine, fourth premolar, and first molar were coupled together as anchorage. In another 2 studies,[7,8] dental implants served as anchorage. One study[5] used prestretched elastics to deliver the predetermined force. These elastics were checked weekly, and, if the force deviated more than 5%, a new elastic was placed. In the other studies,[6-8] superelastic coil springs were used, which have been shown to deliver a constant force over a long activation range.[6]

The 2 studies with tooth anchorage showed a mean anchorage loss of approximately 25%.[5,6] Therefore, in the present study, the rates of tooth movement from these studies were set at 75% of the original ones; anchorage loss in the implant anchorage system was supposed to be zero in the other 2 studies.[7,8] After this correction for anchorage loss, no significant differences were present between the dog studies under identical force conditions, and therefore the data were pooled.

Nine studies on human canines were included.[4,10,12,13,16,17,19,21,26] They were derived from a systematic literature review by our group.[9] The original data from the literature were all recalculated as force magnitude in cN and velocity of tooth movement in millimeters per week before they were entered in the analyses. For studies that reported results only in graphs, velocity was read from the post-lag phase (the linear phase) instead of the tooth movement being calculated over all phases.

When the maxillary and mandibular canines were examined in the same study, no statistically significant difference in the rate of tooth movement was found between them.[13,21] Therefore, data from maxillary and mandibular canines were pooled. Whenever possible, individual data were used. Because in some cases only group means were given, the residual standard deviation will be underestimated. This problem was overcome by adding a normally distributed error to the individual values (means) that had a variance equal to the residual variance. This increases the residual variance, and an iterative procedure leads to the final residual variance. In this way, individual data were simulated. Individual and group data together led to 205 cases.

After the fit of the clinical human data to the dog-derived model had been verified by nonlinear regression analysis (see below), the importance of the covariables, "frequency of reactivation" (no, once per week, twice per week), "type of tooth movement" (bodily or tipping), "duration of the experimental period" (<10 weeks or >10 weeks), and "type of appliance" (helical torsion springs, sectionals, elastics), was analyzed by additional multiple nonlinear regression analyses (see below). The covariables "sex" and "age" of the experimental subjects could not be entered into the analysis because of lack of data.

Model conditions and analyses

The mathematic model was based on the following assumptions: there is no tooth movement without force; the force-velocity curve increases at low forces and reaches a maximum or plateau at higher forces; and the slope of the curve might decrease or remain stable with further increasing forces, but the rate of tooth movement will never become negative (it will never move in a direction opposite to the applied force). These assumptions are fulfilled in a mathematic model according to the equation:

$$V(F) = Vmax \times (F/Fmax)^{Fmax/Fs} \times \exp((Fmax - F)/Fs)$$

in which *V(F)* = the velocity (*V*) as a function of the force (*F*), *Vmax* = the maximal velocity or the velocity at the plateau, and *Fmax* = force at which Vmax is reached. *Fs* = a scaling parameter for the force. The larger the Fs, the less decay in velocity at forces higher than Fmax. A value of Fs close to zero indicates a narrow peak, and a very high value of Fs indicates a plateau for the maximum velocity. A negative Fs implies that the model does not fit to the data. *exp* = the exponential function.

Refinement of the model can be obtained by square-root transformation of the velocity or force. Based on this model, nonlinear regression analyses were applied to the dog data to find the best-fitting square-root transformation.

In a second procedure, the obtained dog model was applied to the clinical data by a comparable nonlinear regression analysis to verify the validity of the model.

The influence of the covariables (effect) was studied by adding a covariable-dependent multiplier (1 + covariable × effect) to the model for V(F). Multiple nonlinear regression estimates the effect of the entered covariable, including its standard error. These effects × 100% can be interpreted as the proportional influence of the corresponding covariable.

RESULTS

The optimal fit for the dog data was obtained after square-root transformation of both velocity and force. The explained variance (R^2) = 43%. The estimated parameters are presented in Table I. The values indicate that the mean maximum velocity of the mandibular second premolar in dogs was 0.27 mm/wk when the force magnitude was 248 cN. Figure 2 shows the force-velocity curve for the model for the dog data with

Table I. Estimated parameters by nonlinear regression in dogs and humans and 95% CIs of V(F)-model after square root transformation and after back transformation

	Mean		*95% CI*	
	Dog	*Human*	*Dog*	*Human*
After root transformation				
Vmax ($(\text{mm/wk})^{1/2}$)	0.52	0.54	0.48–0.55	0.51–0.58
Fmax ($\text{cN}^{1/2}$)	15.75	16.5	10.2–21.3	11.5–21.5
Fs ($\text{cN}^{1/2}$)	50.5	28	6–95	1–55
After back transformation				
Vmax (mm/wk)	0.27	0.29	0.23–0.30	0.26–0.34
Fmax (cN)	248	272	104–454	132–462
Fs (cN)	2550	784	40–9120	2–3080

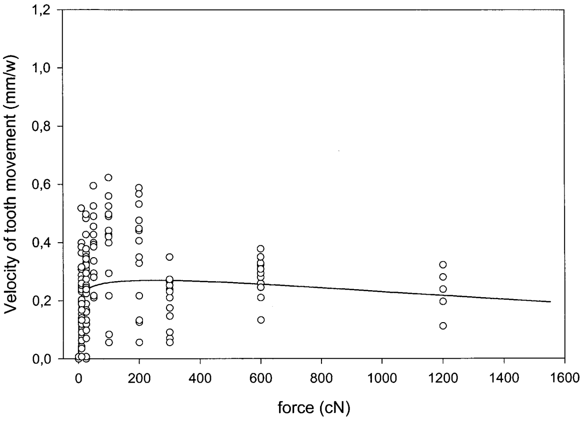

Fig 2. Force–velocity curve representing mathematic model based on dog data; *circles* are individual dog data.

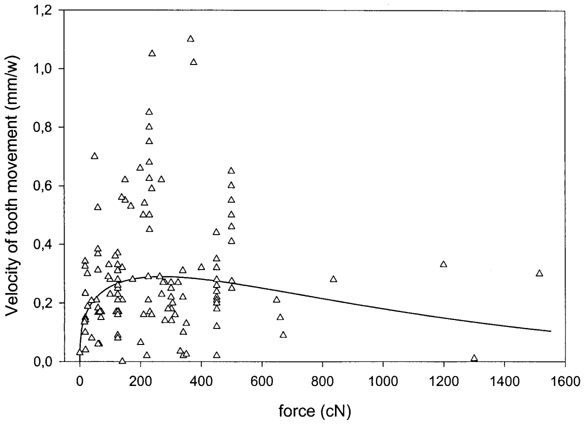

Fig 3. Force–velocity curve presenting result of application of mathematic model derived from dogs to human data; *triangles* are individual or group human data.

the individual data from the literature. The 95% confidence interval (CI) for the scaling parameter Fs is positive in total (Table I). This implies that the model fits well to the data and that the force-velocity curve decreases after Fmax, but, because Fs might be as large as 9120 cN, the curve might also approximate a plateau.

The data for humans were analyzed by nonlinear regression after correcting for group means. The data fit well to the model, because Fs is positive in total but at a low level of explained variance ($R^2 = 6\%$). The results are presented in Table I. The values indicate that the mean maximum velocity of human canine retraction was 0.29 mm/wk when the force magnitude was 272 cN. Figure 3 shows the force-velocity curve for the model for the human data with the individual data from the literature. The 95% CI for Fs shows a wide but positive range (2-3080 cN), which means that the force-velocity curve decreases after Fmax, but that the slope of that part of the curve is not very clear.

The effects of the covariables on the velocity of tooth movement are shown in Table II. The covariables "type of movement" and "duration" had no significant influence on the velocity. On the other hand, "reactivation" and "type of appliance" had a significant effect. If these covariables were taken into account, the explained variance in the model increased from 6% to 22%.

DISCUSSION

Using dogs as a research model for orthodontic tooth movement has many advantages. The differences in the size and anatomy of the periodontal ligament and the alveolar bone between dogs and humans are rather small, although the alveolar bone of dogs is generally thought to be denser than that of humans.[5,27] Furthermore, inbred dogs show little genetic variation; it is rather easy to use a standardized appliance design that produces nearly bodily movement; force magnitude and activation period can be well controlled; and a wide

Table II. Proportional effects of covariables and standard errors (SE) on velocity in clinical data

Covariable	*Effect* ± SE (%)*
Reactivation	
No = 0	−15 ± 4
1/wk = 1	
2/wk = 2	
Type of movement	
Bodily = 0	16 ± 8
Tipping = 1	
Duration	
<10 wk = 0	−1 ± 6
>10 wk = 1	
Type of appliance	
Helical coil spring (A)	A − B: 20 ± 7
Sectionals (B)	C − B: −42 ± 8
Elastics (C)	

*A positive sign of effect indicates that velocity is increased by last-mentioned category of covariables.

range of forces can be applied in dogs without clinical or ethical objections.

The forces used in the current dog model ranged from 10 to 1200 cN; this provided some data for the higher ranges of forces in the force-velocity curve. However, even with this information, it seems impossible to draw a conclusion on the slope of the curve at high force levels, because of the wide 95% CI for Fs.

The combined experimental data from 4 dog studies[5-8] were fit to a mathematic equation by which the mean maximum rate of tooth movement could be predicted with a 95% CI of 0.23 to 0.30 mm/wk. The optimum force magnitude derived from this equation showed a more substantial range (95% CI 104-454 cN). This suggests that force magnitude is not the major decisive factor for the rate of tooth movement and that any force within that range might evoke the required biologic response for optimal tooth movement in the periodontal tissues.

The choice to use the same mathematic approach for human data as for the dog data is based on the general assumption that the gross characteristics of the relationship between force and rate of tooth movement are similar in both species. Both curves presumably will go through the origin (0,0) and will show a dose-response relationship at low forces; the velocity will reach a maximum or plateau, and it probably will decrease again with increasing forces, but the velocity will never become negative.

The nonlinear regression analyses, which were performed separately for dog and human data, showed that the best fits for dogs and humans were reached when estimates for Vmax and Fmax were close together. The best estimate for Vmax was calculated to be 0.27 mm/wk for dog second premolars and 0.29 mm/wk for human canines; this indicates that there is no clear difference between the 2. This supports the idea that the rate of bone turnover that can be induced by mechanical stress is not species-specific. The best estimate for Fmax for dog premolars was calculated to be 248 cN, whereas it was 272 cN for human canines. These differences, however, are not significant: the 95% CIs are 104-454 cN and 132-462 cN, respectively.

This result is surprising at first sight, because the surface area of the roots of beagle second mandibular premolars is approximately 1 cm^2 and that of a human canines approximately 2.9 cm^2.[5,28] This means that the resulting pressure caused by a certain force on a dog premolar can be considered to be about 3 times as high as the pressure that it causes on a human canine.

An explanation might be that, in most human canine studies, tipping forces were used, whereas, in the dog studies, bodily tooth movement was performed. These different types of movement are related to differences in stress distribution in the periodontal ligament and therefore might be related to different biologic responses. However, statistical analysis of "type of movement" as a covariable showed that it had no significant effect. Another explanation could be that the range of force magnitudes that induces maximal biologic response is larger than could be estimated from the present data and that additional data, especially in the high force range, are needed to improve the accuracy of the estimate.

Statistical analysis of the covariables indicates that increasing the frequency of reactivation leads to a decrease in the rate of tooth movement. Also, the type of appliance has an effect on the velocity. Coil springs result in a lower rate, and elastics in a higher rate, of movement than sectionals. Although the explained variance increases to 22% if the covariables are taken into account, the explanation of these effects is difficult, owing to possible interactions and confounding factors.

Comparison of our data with the models described by Quinn and Yoshikawa[1] shows that none of their models is properly supported by our data. Our study showed that a dose-response relationship exists in the very low force range; then a plateau is reached that can be considered as representative for the optimal force range. There is a tendency for the tooth movement to slow down when the force increases, but the slope of that curve cannot be predicted, as is indicated by the large 95% CI of Fs. Therefore, it remains unclear at which force the movement will cease completely, as is predicted by the third Quinn and Yoshikawa model (Fig 1, *C*). The other 3 models (Fig 1, *A*, *B*, *D*) show some principal differences from ours: they predict either a constant rate of tooth movement or a continuous increase with increasing forces.

Moreover, all the Quinn and Yoshikawa models assume that the force should exceed a certain threshold before the system is switched on, whereas in our model tooth movement is also possible with minute forces. This indicates that very small changes in pressure can induce the biologic responses necessary for the induction of bone turnover. A clinical consequence of these results is that it is virtually impossible to control anchorage. Very low forces, as well as forces beyond the optimum level, will lead to substantial tooth movement.

The present study shows that well-controlled clinical trials and animal experiments are needed to provide better insight into the relationship between orthodontic forces and rate of subsequent tooth movement. The model proposed in our study also needs to be refined by more complete data because the lack of data on high forces hinders its validation. Because clinical and ethical considerations limit the possibility of applying high forces on humans for research purposes, animal studies are indicated, and beagles could be a very good model. The results of the present study are apparently not suited to predict the force-velocity relationship in an individual case because no simple mathematic relationship seems to exist between force magnitude and rate of tooth movement. The validity of the model could probably be improved if more data from controlled experimental studies using very low or very high forces were available. However, the data presented in this study provide a general description of the relationship between orthodontic force and the subsequent rate of tooth movement, and this model can be used as a theoretic framework for future studies.

CONCLUSIONS

This study has shown that the maximum rates of tooth movement in humans and dogs are not significantly different. It has also shown that, from data in the current literature, no threshold can be defined for the force or, more accurately, the increase in pressure that will switch on tooth movement, nor can an optimal force or force range be calculated that produces maximum tooth movement.

A minute force, leading to a minute change in pressure, might be able to switch on tooth movement. This implies that higher forces often used in orthodontic practice do not necessarily produce more efficient tooth movement. On the contrary, they might overload the periodontal tissues and cause negative effects that will hinder tooth movement.

REFERENCES

1. Burstone CJ. The biophysics of bone remodeling during orthodontics—optimal force considerations. In: Norton LA, Burstone CJ, editors. The biology of tooth movement. Boca Raton: CRC Press; 1989. 321-34.
2. Quinn RS, Yoshikawa K. A reassessment of force magnitude in orthodontics. Am J Orthod 1985;88:252-60.
3. Owman-Moll P, Kurol J, Lundgren D. Effects of a doubled orthodontic force magnitude on tooth movement and root resorption. An inter-individual study in adolescents. Eur J Orthod 1996;18:141-50.
4. Iwasaki L, Haack JE, Nickel JC, Morton J. Human tooth movement in response to continuous stress of low magnitude. Am J Orthod Dentofacial Orthop 2000;117:175-83.
5. Pilon JJ, Kuijpers-Jagtman AM, Maltha JC. Magnitude of orthodontic force and rate of bodily tooth movement, an experimental study. Am J Orthod Dentofacial Orthop 1996;107:16-23.
6. Van Leeuwen EJ, Maltha JC, Kuijpers-Jagtman AM. Tooth movement with light continuous and discontinuous forces in beagle dogs. Eur J Oral Sci 1999;107:468-74.
7. Von Böhl M, Maltha JC, Von den Hoff JW, Kuijpers-Jagtman AM. Focal hyalinization limits experimental tooth movement in "slow movers" [abstract]. Eur J Orthod 2001;23:468.
8. Maltha JC, Van Leeuwen EJ, Van 't Hof MA, Kuijpers-Jagtman AM. Effect of changing force magnitude on the rate of orthodontic tooth movement [abstract]. Eur J Orthod 2001;23:458.
9. Ren Y, Maltha JC, Kuijpers-Jagtman AM. Optimum force magnitude for orthodontic tooth movement: a systematic literature review. Angle Orthod 2003;73:86-92.
10. Storey E, Smith R. Force in orthodontics and its relation to tooth movement. Aust Dent J 1952;56:11-8.
11. Andreasen GF, Zwanziger D. Experimental findings on tooth movement under two conditions of applied force. Angle Orthod 1967;37:9-12.
12. Hixon EH, Atikian H, Callow GE, McDonald HW. Optimal force, differential force, and anchorage. Am J Orthod 1969;55:437-57.
13. Hixon EH, Aasen TO, Arango J, Clark RA, Klosterman R, Miller SS, et al. On force and tooth movement. Am J Orthod 1970;57:476-89.
14. Owman-Moll P, Kurol J, Lundgren D. The effects of a four-fold increased orthodontic force magnitude on tooth movement and root resorption. An intra-individual study in adolescents. Eur J Orthod 1996;18:287-94.
15. Begg PR. Differential force in orthodontic treatment. Am J Orthod 1956;42:481-510.
16. Lee BW. Relationship between tooth-movement rate and estimated pressure applied. J Dent Res 1965;44:1053.
17. Lee BW. The force requirements for tooth movement. Part 1: tipping and bodily movement. Aust Orthod J 1995;13:238-48.
18. Burstone CJ, Groves MH. Threshold and optimum force values for maxillary tooth movement. J Dent Res 1961;39:695-6.
19. Andreasen GF, Zwanziger D. A clinical evaluation of the differential force concept as applied to the edgewise bracket. Am J Orthod 1980;78:25-40.
20. Keeling SD, King GJ, McCoy EA, Valdez M. Serum and alveolar bone phosphatase changes reflect bone turnover during orthodontic tooth movement. Am J Orthod Dentofacial Orthop 1993;103:320-6.
21. Boester CH, Johnston LE. A clinical investigation of the concept

of differential and optimal force in canine retraction. Angle Orthod 1974;44:113-9.

22. Berg R. Intraoral measurement on the pattern of initial orthodontic tooth movement. Scand J Dent Res 1970;78:453-61.
23. King GJ, Keeling SD, McCoy EA, Ward TH. Measuring dental drift and orthodontic tooth movement in response to various forces in adult rats. Am J Orthod Dentofacial Orthop 1991;99: 456-65.
24. Steiner GG, Pearson JK, Ainamo J. Changes of the marginal periodontium as a result of labial tooth movement in monkeys. J Periodontol 1981;52:314-20.
25. Fortin JM. Translation of premolars in the dog by controlling the moment-to-force ratio on the crown. Am J Orthod 1971;59:541-50.
26. Daskalogiannakis J, McLachlan KR. Canine retraction with rare earth magnets: an investigation into the validity of the constant force hypothesis. Am J Orthod Dentofacial Orthop 1996;109: 489-95.
27. Reitan K, Kvam E. Comparative behavior of human and animal tissue during experimental tooth movement. Angle Orthod 1971; 41:1-14.
28. Hujoel PP. A meta-analysis of normal ranges for root surface areas of the permanent dentition. J Clin Periodontol 1994;21: 225-9.